

On the geometry of the space-time and motion of the spinning bodies

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Abstract

In this paper an alternative theory about space-time is given. First some preliminaries about 3-dimensional time are presented, and the reasons for the introducing of the 3-dimensional time are also given. Beside the 3-dimensional space (S) it is considered the 3-dimensional space of spatial rotations (SR) independently from the 3-dimensional space. Then it is given a model of the universe, based on the Lie groups of real and complex orthogonal 3×3 matrices in this 3+3+3-dimensional space. Special attention is dedicated to introduction and study of the group over $S \times SR$, which appears to be isomorphic to $SO(3, \mathbb{R}) \times SO(3, \mathbb{R})$. From viewpoint of the ordinary geometry this is analogous to the affine group of all translations and rotations in 3-dimensional Euclidean space, which is homeomorphic to $SO(3, \mathbb{R}) \times \mathbb{R}^3$. Some important applications of these results about spinning bodies are given, which naturally lead to violation of the third Newton's law, the law of preserving the total energy and the Principle of Equivalence from the General Relativity and gives a generalization of the space-time element known from the Special Relativity. At the end of the paper are considered spinning bodies in gravitational field and their departures from the non-spinning motion, which may easily be experimentally verified.

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1 Introduction

The Special and the General Relativity give the geometry of the 3+1- dimensional space-time. The Spacial Relativity is free of any anomaly, if we assume that the space-time is homeomorphic to \mathbb{R}^4 . The General Relativity appears to be extremely good when we consider trajectories of motion, gravitational radiation and so on. But if we consider the precession of axis of a gyroscope, we have a different situation. Using the Fermi-Walker connection, it is well

known that the angular velocity of the axis of a gyroscope in a free fall orbit is given by ([17], eq. (9.5))

$$\vec{\Omega} = \left(\gamma + \frac{1}{2}\right) \sum_a (\vec{v} - \vec{v}_a) \times \nabla \frac{Gm_a}{r_a c^2} - \frac{1}{2}(\gamma + 1) \sum_a G[\vec{J}_a - 3\hat{n}_a(\hat{n}_a \cdot \vec{J}_a)]/r_a^3 c^2 - \frac{1}{2} \sum_a \vec{v}_a \times \nabla \frac{Gm_a}{r_a c^2}, \quad (1)$$

where \vec{v} is the velocity of the gyroscope, \vec{v}_a is the velocity of the a -th spherical body, \vec{J}_a is its angular momentum and r_a is its distance to the gyroscope. The third term is anomalous since it depends on the velocity of each body [17]. Although there is an effort to explain why experimentally this term can not be observed, or it leads to a small periodic effect in case of an observation as the Gravity Probe B experiment, it remains to be an anomaly. In the paper [15] this anomaly is solved by assuming axiomatically a precession of the coordinate system, i.e. coordinate axes. This precession is analogous to the Thomas precession, which is related to precession of the gyroscope, and so we will call it Thomas precession too. The final conclusion is that observed close from the gyroscope the precession of the gyroscope's axis is given by

$$\vec{\Omega}_{gyr.} = 2 \sum_a (\vec{v} - \vec{v}_a) \times \nabla \frac{Gm_a}{r_a c^2} - \sum_a G[\vec{J}_a - 3\hat{n}_a(\hat{n}_a \cdot \vec{J}_a)]/r_a^3 c^2, \quad (2)$$

the observed apparent (not true) precession of the distant stars, which is a consequence of the precession of the coordinate system, is given by

$$\vec{\Omega}_{stars} = \frac{1}{2} \sum_a (\vec{v} - \vec{v}_a) \times \nabla \frac{Gm_a}{r_a c^2} - \frac{1}{4} \sum_a G[\vec{J}_a - 3\hat{n}_a(\hat{n}_a \cdot \vec{J}_a)]/r_a^3 c^2, \quad (3)$$

and hence its subtraction, i.e. the relative precession of the gyroscope's axis with respect to the distant stars, is given by

$$\vec{\Omega}_{rel.} = \frac{3}{2} \sum_a (\vec{v} - \vec{v}_a) \times \nabla \frac{Gm_a}{r_a c^2} - \frac{3}{4} \sum_a G[\vec{J}_a - 3\hat{n}_a(\hat{n}_a \cdot \vec{J}_a)]/r_a^3 c^2. \quad (4)$$

All these precessions are Lorentz covariant angular velocities. The rotation of the coordinate axes is necessary in order to obtain a Lorentz covariant result. But the precession (3) is also necessary to be applied if we want to obtain Lorentz covariant results about the equations of motion, for some special choices of the observer.

The precession of the coordinate axes, which is opposite to (3), is such that the observer does not feel any inertial force in his neighborhood. For example he can not detect the Coriolis force in his neighborhood.

Notice that according to (4) the geodetic precession with respect to the distant stars is the same as it is well known from the General Relativity and as it is experimentally confirmed by the Gravity Probe B experiment and precession

of the system Earth-Moon as a gyroscope around the Sun. But the frame dragging is 25% less than the known values from the General Relativity. Notice that these 25% can not be detected via the Lense-Thirring effect which arises from the equations of motion, while the frame dragging effect arises from the Fermi-Walker connection. So the Gravity Probe B is unique experiment where these 25% would be measurable. But unfortunately, the large uncertainties in this experiment do not permit precise value for the frame dragging [6].

The mentioned anomaly appears as a consequence of the fact that the space-time is a priori assumed to be 3+1-dimensional. In [15] the problem is solved and equations (2), (3) and (4) are deduced

- from the following axiom "An observer who rests with respect to a non-rotating gravitational body observes no precession of the coordinate axes of any freely moving coordinate system" and

- accepting the known formula for the geodetic precession $\frac{3}{2} \sum_a (\vec{v} - \vec{v}_a) \times \nabla \frac{Gm_a}{r_a c^2}$ from (4) with respect to the distant stars, which is experimentally confirmed.

But on the other side, since all calculations in differential geometry, General Relativity and tensor calculus are done with respect to the chosen coordinate system, but not with respect to the distant stars, it is natural to expect that the geodetic precession $2 \sum_a (\vec{v} - \vec{v}_a) \times \nabla \frac{Gm_a}{r_a c^2}$ from (2) should be derived in local coordinate system. But this is not a case in the frame of the General Relativity. This is the second anomaly.

Although the first anomaly was solved inside the 3+1-dimensional space-time, the second anomaly can not be solved at the same time. It is solved (in a recent unpublished paper) by researching gravitation in 3+3-dimensional space-time. The main feature there is that it is not considered parallel transport of vectors, but parallel transport of Lorentz transformations. Hence the anomalies with the precession of the gyroscope's axis are solved and the same approach ([16, 12]) also it fits with all of the gravitational experiments which are performed for verification of the General Relativity.

After all these arguments we see that the space-time is not so simple, and it requires answer of some basic questions about the space, time, velocities and motions. Such questions will be subject of this paper¹.

We convenient in this paper to use the temporal coordinate as ict , instead of ct .

2 3-dimensional time

In this section we give some preliminaries in condensed form, about the concept of the 3-dimensional time [13]. The old concept of time is indeed 1-dimensional parameter in the high dimensions, which will be described in this section.

We mentioned in the previous section that it is better to consider parallel transport of a Lorentz transformation instead of parallel transport of a 4-vector

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of velocity. The result is not the same, because the parallel transport of a 4-vector of velocity is always a 4-vector, while the parallel transport of a Lorentz transformation which is a Lorentz boost at the initial moment, i.e. without space rotation, may not be Lorentz boost, because may contain space rotation too. In case of parallel transport of a 4-vector of velocity, the consequence is appearance of anomalies, which were described in the previous section. This is the main motivation to research a model of 3-dimensional time. Albert Einstein and Henri Poincare many years ago thought about 3-dimensional time, such that the space and time would be of the same dimension. At present time most of the authors [18, 9, 1, 2, 3, 4, 5, 7, 8, 10, 11] propose multidimensional time in order to give better explanation of the quantum mechanics and the spin.

Let us denote by x , y and z the coordinates in our 3-dimensional space \mathbb{R}^3 , and let us consider the principal bundle over the base \mathbb{R}^3 , whose fiber is $SO(3, \mathbb{C})$ and the (complex) Lie group is $SO(3, \mathbb{C})$ too. This bundle will be called *space-time bundle*. Having in mind that the unit component $O_+^\uparrow(1, 3)$ of the Lorentz group is isomorphic to $SO(3, \mathbb{C})$, this bundle can be considered simply as the bundle of all moving orthonormal Lorentz frames. The space-time bundle locally can be parameterized by the following 9 local coordinates $\{x, y, z\}$, $\{x_s, y_s, z_s\}$, $\{x_t, y_t, z_t\}$, such that the first 6 coordinates parameterize locally the subbundle with the fiber $SO(3, \mathbb{R})$. The local coordinates x_s, y_s, z_s are called *spatial coordinates*, while x_t, y_t, z_t are called *temporal coordinates*. So this approach in the Special Relativity is called 3+3+3-dimensional model. Indeed, to each body are related 3 coordinates for the position, 3 coordinates for the space rotation and 3 coordinates for its velocity.

The analog of the Lorentz boosts from the 3+1-dimensional space-time is indeed the set of Hermitian matrices from the complex Lie group $SO(3, \mathbb{C})$. Arbitrary Hermite matrix from $SO(3, \mathbb{C})$ can uniquely be written in the following form $\cos A + i \sin A$, for a corresponding antisymmetric matrix A . This matrix can be parameterized with a 3-vector of velocity $\vec{v} = (v_x, v_y, v_z)$ ($|\vec{v}| < c$), such that

$$\sin A = \frac{-1}{c\sqrt{1 - \frac{v^2}{c^2}}} \begin{bmatrix} 0 & v_z & -v_y \\ -v_z & 0 & v_x \\ v_y & -v_x & 0 \end{bmatrix}, \quad (5)$$

and

$$(\cos A)_{ij} = V_4 \delta_{ij} + \frac{1}{1 + V_4} V_i V_j, \quad (6)$$

where $(V_1, V_2, V_3, V_4) = \frac{1}{ic\sqrt{1 - \frac{v^2}{c^2}}} (v_x, v_y, v_z, ic)$.

Now let us consider the following mapping $F : O_+^\uparrow(1, 3) \rightarrow SO(3, \mathbb{C})$ given

by

$$\begin{bmatrix} M & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 - \frac{1}{1+V_4} V_1^2 & -\frac{1}{1+V_4} V_1 V_2 & -\frac{1}{1+V_4} V_1 V_3 & V_1 \\ -\frac{1}{1+V_4} V_2 V_1 & 1 - \frac{1}{1+V_4} V_2^2 & -\frac{1}{1+V_4} V_2 V_3 & V_2 \\ -\frac{1}{1+V_4} V_3 V_1 & -\frac{1}{1+V_4} V_3 V_2 & 1 - \frac{1}{1+V_4} V_3^2 & V_3 \\ -V_1 & -V_2 & -V_3 & V_4 \end{bmatrix} \mapsto M \cdot (\cos A + i \sin A), \quad (7)$$

where $\cos A$ and $\sin A$ are given by (6) and (5). This is well defined because the decomposition of any matrix from $O_+^\uparrow(1, 3)$ as product of a space rotation and a boost is unique. Moreover, it is an isomorphism between the two groups and the corresponding isomorphism between their Lie algebras is given by

$$\begin{bmatrix} 0 & c & -b & ix \\ -c & 0 & a & iy \\ b & -a & 0 & iz \\ -ix & -iy & -iz & 0 \end{bmatrix} \mapsto \begin{bmatrix} 0 & c + iz & -b - iy \\ -c - iz & 0 & a + ix \\ b + iy & -a - ix & 0 \end{bmatrix}.$$

The isomorphism between the groups $O_+^\uparrow(1, 3)$ and $SO(3, \mathbb{C})$ is the main reason why we observe our space-time as 4-dimensional where the group $O_+^\uparrow(1, 3)$ acts on it, instead of 3+3-dimensional space-time.

We notice that a rotation for an imaginary angle denotes a motion with a velocity. But if we want to deduce the Lorentz transformations, we must consider change of the basic coordinates x, y, z . Hence we assume that the spatial coordinates x_s, y_s, z_s and temporal coordinates x_t, y_t, z_t are functions of the basic coordinates, such that the corresponding Jacobi matrices satisfy

$$\left[\frac{\partial(x_s, y_s, z_s)}{\partial(x, y, z)} \right] = \cos A, \quad \left[\frac{\partial(x_t, y_t, z_t)}{\partial(x, y, z)} \right] = \sin A, \quad (8)$$

where $\cos A$ and $\sin A$ in case of Lorentz boost with velocity \vec{v} were previously determined. From the equalities (8) and (5) the time vector in this special case is given by

$$(x_t, y_t, z_t) = \frac{\vec{v}}{c\sqrt{1 - \frac{v^2}{c^2}}} \times (x, y, z) + (x_t^0, y_t^0, z_t^0), \quad (9)$$

where (x_t^0, y_t^0, z_t^0) does not depend on the basic coordinates. The coordinates x_t, y_t, z_t are independent, while the Jacobi matrix $\left[\frac{\partial(x_t, y_t, z_t)}{\partial(x, y, z)} \right]$ is a singular matrix as antisymmetric matrix of order 3, where the 3-vector of velocity \vec{v} maps into zero vector. So the quantity $(x_t, y_t, z_t) \cdot \vec{v}$ does not depend on the basic coordinates and so we assume that *it is proportional to the 1-dimensional*

time perimeter t , measured from the basic coordinates. The formula (9) can be written also in the following form

$$(x_t, y_t, z_t) = \frac{\vec{v}}{c} \times (x_s, y_s, z_s) + \vec{c} \cdot \Delta t, \quad (10)$$

where \vec{c} is the velocity of light, which has the same direction as \vec{v} , i.e. $\vec{c} = \frac{\vec{v}}{v} \cdot c$.

According to the moving system, x_s, y_s, z_s may be basic coordinates, and then according to this base, the coordinates x, y, z appear to be spatial coordinates, and let the corresponding temporal coordinates are ct_x, ct_y, ct_z . Hence we have two complex coordinates $(x_s + ix_t, y_s + iy_t, z_s + iz_t)$ and $(x + ict_x, y + ict_y, z + ict_z)$. These coordinates are related in the following way. Let the coordinates x_s, y_s, z_s are denoted by x', y', z' and let us denote $\vec{r} = (x, y, z)$ and $\vec{r}' = (x', y', z')$. In the recent paper [14] it is proved that the following transformation in \mathbb{C}^3

$$\left(1 - \frac{v^2}{c^2}\right)^{-1/2} \begin{bmatrix} \Delta \vec{r}_s \\ \Delta \vec{r}_t \end{bmatrix} = \begin{bmatrix} \cos A & -\sin A \\ \sin A & \cos A \end{bmatrix} \begin{bmatrix} \Delta \vec{r} + \vec{v}(t + \delta t) \\ \vec{c}(t + \delta t) \end{bmatrix} \quad (11)$$

via the group $SO(3, \mathbb{C})$ is equivalent to the transformation given by a Lorentz boost determined by the isomorphism (7).

Notice that in (11) there exists a translation in the basic coordinates for vector $(\vec{v}(t + \delta t), \vec{c}(t + \delta t))$, where $\delta t = \frac{\vec{r}' \cdot \vec{v}}{\sqrt{1 - \frac{v^2}{c^2}}}$ appears from the non-simultaneity of the start point and the endpoint analogously as in the Special Relativity. The coefficient $\beta = (1 - \frac{v^2}{c^2})^{-1/2}$ appears from the fact that the components $\Delta \vec{r}_s$ and $\Delta \vec{r}_t$ are measured from the basic coordinates. If they are observed from the self coordinate system, then (11) becomes

$$\begin{bmatrix} \Delta \vec{r}_s \\ \Delta \vec{r}_t \end{bmatrix} = \begin{bmatrix} \cos A & -\sin A \\ \sin A & \cos A \end{bmatrix} \begin{bmatrix} \Delta \vec{r} + \vec{v}(t + \delta t) \\ \vec{c}(t + \delta t) \end{bmatrix}. \quad (12)$$

We notice that the well known 4-dimensional space-time is not fixed in 6 dimensions, but changes with the direction of velocity. Namely this 4-dimensional space-time is generated by the basic space vectors and the velocity vector from the imaginary part of the complex base.

Analogously as the quantity $(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2$ is invariant for the group of transformations $SO(3, \mathbb{R})$ if x, y and z are real numbers, the quantity $(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2$ is invariant, for the group of transformations $SO(3, \mathbb{C})$, where x, y and z are complex numbers. Hence, we have now two invariants:

$$Re[(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2] \text{ and } Im[(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2].$$

We assume that the first invariant is positive or zero, while the second invariant $Im[(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2]$ is zero for simultaneous initial point and end point ([13]).

3 Introduction to the 3+3+3-model of cosmology

Now let us introduce a possible global construction of the universe. At the beginning of the previous section we considered the space-time bundle with base B and fiber $SO(3, \mathbb{C})$. The group $SO(3, \mathbb{C})$ is much convenient because it can be considered simultaneously as a fiber and Lie group. This is not case of its isomorphic group $O_+^\uparrow(1, 3)$, which acts on 4-dimensional space-time. For the sake of simplicity we assumed that $B = \mathbb{R}^3$, but the basic space topologically should be closed and bounded as it is accepted and so it should be changed. On the other side the equality (12) suggests that the space-time (without rotations) is a complex manifold. Indeed, analogously as $(x + ict_x, y + ict_y, z + ict_z)$ is a local coordinate neighborhood of $SO(3, \mathbb{C})$, $(x_s + ix_t, y_s + iy_t, z_s + iz_t)$ is also a coordinate neighborhood of the same manifold. These two coordinate systems can be considered as coordinate neighborhoods of $SO(3, \mathbb{C})$ as a complex manifold, because according to (12) the Cauchy-Riemannian conditions for these two systems are satisfied.

If we assume that the basic space is homeomorphic to RP^3 , i.e. $SO(3, \mathbb{R})$, we come to satisfactory result. The complexification of this group is $SO(3, \mathbb{C})$ and it is the same as the fiber and the Lie group of the space-time bundle. The local coordinates of $SO(3, \mathbb{R})$ are angles, i.e. real numbers, but we use length units for our local spatial coordinates. So for each small angle φ of rotation in a given direction corresponds coordinate length $R\varphi$ in the same direction, where R is a constant, which can be called *radius of the universe*.

We give now some additional reasons for this assumption. The topological space $SO(3, \mathbb{R})$ admits 3 linearly independent vector fields, which are orthogonal at each point, because each n -dimensional real Lie group admits n linearly independent vector fields. Indeed we can choose n vector fields at the unit and then we may parallel transport using the group structure to any other point. Each Lie group admits a connection with zero curvature, but non-zero torsion tensor in general case. The previously mentioned three vector fields may be chosen to be orthonormal at each point, and they are parallel with respect to the mentioned connection with zero curvature. This is in accordance with the recent astronomical observations, which show that our universe is a flat (non-curved) space.

Let us discuss the space-time dimensionality of the universe. We have that it is parameterized by the following 9 independent coordinates: x, y, z coordinates which locally parameterize the spatial part of the universe $SO(3, \mathbb{R})$, and $x_s, y_s, z_s, x_t, y_t, z_t$ coordinates which parameterize the bundle. Also, the partial derivatives of these coordinates with respect to x, y, z lead to the same manifold, but now as a group of transformations. So in any case the total space-time of the universe is homeomorphic to $SO(3, \mathbb{R}) \times SO(3, \mathbb{C})$, i.e. $SO(3, \mathbb{R}) \times \mathbb{R}^3 \times SO(3, \mathbb{R})$. In the above parameterization, \mathbb{R}^3 is indeed the space of velocities such that $|\vec{v}| < c$. The group $SO(3, \mathbb{C})$, as well as its iso-

morphic group $O_+^\uparrow(1, 3)$, considers only velocities with magnitude less than c . If $|\vec{v}| = c$, then we have a singularity.

Notice that if we consider that the universe is a set of points, then it is natural to consider it as 6-dimensional. But in such 6-dimensional space-time, rotations will not be admitted, because the existence of the 3-dimensional space does not mean that the space rotations are also admitted. Analogously, if we neglect the three temporal coordinates, the motions will not be admitted. But, since we consider the universe as a set of orthonormal moving frames, so it is more natural to consider it as a 9-dimensional.

This 9-dimensional space-time has the following property: From each point of the space, each velocity and each spatial direction of the observer, the universe seems to be the same. In other words, assuming that R is a global constant, there are no privileged spatial points, no privileged directions and no privileged velocities.

The previous discussion leads to the following diagram,

$$\begin{array}{ccc} & & V \cong \mathbb{R}^3 \\ & \times & \times \\ S \cong SO(3, \mathbb{R}) & \times & SR \cong SO(3, \mathbb{R}) \end{array}$$

consisting of three 3-dimensional sets: velocity (V) which is homeomorphic to \mathbb{R}^3 , space (S) which is homeomorphic to $SO(3, \mathbb{R})$, and space rotations (SR) which is homeomorphic to $SO(3, \mathbb{R})$.

4 Research of the 6-dimensional space $SR \times S$

In the previous section some preliminaries were given. In this section and also in the following sections we will investigate the Lie groups in the 3+3+3-model and their invariants. We shall see that there is a symmetry between the sets S and SR .

We almost know that the product $SR \times V \cong SO(3, \mathbb{C})$ can be considered as a fiber and Lie group of a trivial principal bundle with base S . So we consider this group for a fixed basic point in S . The Lie group $SO(3, \mathbb{C})$ will be denoted by G_t , with index t (time). This principal bundle is close to the methods of the classical mechanics. Indeed, it is sufficient to study the law of the change of the matrices from the fiber, i.e. the matrices which consist the informations about the spatial rotation and the velocity vector of a considered test body. Then it is easy to find the trajectory of motion of the test body.

Now let us consider the product $S \times SR$. The product $S \times SR$ can be considered as a fiber and Lie group of a trivial principal bundle over the base V . So we consider this group for a fixed temporal point in V , i.e. for a fixed inertial coordinate system up to a space translation and space rotation. So the coefficient $\sqrt{1 - \frac{v^2}{c^2}}$ will not be applied. It doesn't mean that 1-parametric time does not change, but simply the velocities may be very slow and neglected.

The structure Lie group will be denoted by G_s with index s (space), and this group should be analogous to the group of all rotations and translations in the 3-dimensional Euclidean space. On the other side this group is analogous to the Lorentz group G_t . While the Lie algebra of G_t has the form

$$\begin{bmatrix} C & B \\ -B & C \end{bmatrix}, \quad (13)$$

the Lie algebra of G_s is given by

$$\begin{bmatrix} C & B \\ B & C \end{bmatrix}. \quad (14)$$

Here C and B are antisymmetric 3×3 matrices, where the matrix C is the Lie algebra which corresponds to the space rotations, i.e. to the Lie algebra of $SO(3, \mathbb{R})$, while B is an antisymmetric matrix which corresponds to the Lie algebra of S , if we identify S with the Lie group $SO(3, \mathbb{R})$. So, if we put

$$C = \begin{bmatrix} 0 & -c_3 & c_2 \\ c_3 & 0 & -c_1 \\ -c_2 & c_1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & -b_3 & b_2 \\ b_3 & 0 & -b_1 \\ -b_2 & b_1 & 0 \end{bmatrix},$$

(c_1, c_2, c_3) determines a 3-vector for a small space rotation, while (b_1, b_2, b_3) is proportional to the 3-vector of angular rotation in S . Practically, since the radius of the universe R is extremely large, the components b_1, b_2 and b_3 are proportional to $1/R$ and so they are very small, and $R(b_1, b_2, b_3)$ can be considered as a vector of translation.

It is easy to verify that the mapping

$$\begin{bmatrix} C & B \\ B & C \end{bmatrix} \mapsto (C + B, C - B),$$

defines an isomorphism between the Lie algebra of G_s and the Lie algebra of $SO(3, \mathbb{R}) \times SO(3, \mathbb{R})$. Note that $B \mapsto -B$ gives an automorphism of the group G_s . These comments support our assumption from section 3 that S is homeomorphic to $SO(3, \mathbb{R})$, such that $S \times SR$ is really homeomorphic to $SO(3, \mathbb{R}) \times SO(3, \mathbb{R})$.

While the matrices from the group $G_t = SO(3, \mathbb{C})$ can be interpreted in the 3+1-dimensional space-time, the matrices from the group G_s do not have such interpretation in 3+1-dimensional space-time. But if we neglect the terms of order $1/R^2$, the group G_s reduces to the group of all rotations and translations in the Euclidean space, i.e. all matrices of type $\begin{bmatrix} M & \vec{h}^T \\ 0 & 1 \end{bmatrix}$, where

$M \in SO(3, \mathbb{R})$ and \vec{h}^T is the vector of translation. Indeed, the corresponding mapping for their Lie algebras appears to be approximately an isomorphism (neglecting $1/R^2$) between the corresponding Lie algebras. Analogously if we neglect c^{-2} , then the group G_t reduces to the group of Galilean transformations.

We will determine the set of matrices in G_s in the following way. We know that any matrix from G_t can be written as product of a Lorentz boost and a space rotations, i.e. matrix of form $\begin{bmatrix} M & 0 \\ 0 & M \end{bmatrix}$, where M determines a space rotation, and conversely, as a product of a space rotation and a Lorentz boost. Analogously can be proved to be true also for the group G_s . So we need only to determine the matrices which are analogous to the Lorentz boosts. This subgroup of transformations which correspond to the "translations" in the z -direction are given by matrices of the form

$$\begin{bmatrix} \cos \alpha & 0 & 0 & 0 & \sin \alpha & 0 \\ 0 & \cos \alpha & 0 & -\sin \alpha & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & \sin \alpha & 0 & \cos \alpha & 0 & 0 \\ -\sin \alpha & 0 & 0 & 0 & \cos \alpha & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \quad (15)$$

Although G_s is isomorphic to the Cartesian product $SO(3, \mathbb{R}) \times SO(3, \mathbb{R})$, the two multipliers in this isomorphism are not S and SR .

This principal bundle will have important applications in section 5. If a *solid body* is spinning, there may appear a constraint for the space rotation, but no constraint for the place position of the moving frame. Before we consider it in more details, we give examples when these constraints occur.

Example 1. Assume that we have a solid circle which rotates around its axis. Then it appears a constraint for the particles of the circle, because the circle is a solid body and the space rotation for all particles must be the same. So the Thomas precession which appears for particles of the circle can not be realized.

Example 2. While the previous example leads to a negligible relativistic effect, this example gives easy observable effects. We know that the spinning bodies, like coins, footballs and so on, where the gyroscope's axis is not constant, show some departures from the classical physical laws, for example the angular momentum is not preserved. A spinning football, which moves on a free-fall orbit under the Earth's gravitation shows deviations from the parabolic orbit. A spinning coin moves on a circle and the law of inertia does not hold in this case.

In case of a constraint, the 3-vector of unadmitted angular rotation will be converted into space displacement by vector multiplication with the corresponding vector \vec{r} , where the center of the vector \vec{r} is the center of the osculatory circle. This is general conclusion and later in section 6 it will be supported

by the 6-dimensional space $S \times SR$ with the structural group G_s . This property of conversion from unadmitted space rotation into space displacement will be called *basic property of the space*. This displacement in the space which is a result of the basic property of the space will be called *spin displacement*. Its first and second derivative by the 1-parametric time t will be called *spin velocity* and *spin acceleration* respectively and will be denoted by large letters V and A respectively. While the ordinary velocity has the property of inertia and it is limited by the light velocity c , the spin velocity is non-inertial and it is not limited, because it can be conceived just like a displacement. But practically, if a non-spinning mass is attached to the rotating body, then this non-spinning mass has inertia and hence the basic spin velocity is limited by c .

5 Application to rotating bodies

First we shall study a rotating sphere with radius R . This is uniquely determined by a family of orthogonal matrices $A(t) \in SO(3, \mathbb{R})$, where t is 1-dimensional time parameter. At each moment each point of the sphere moves with a tangent 3-vector of velocity. This vector field is a continuous, and so there exists at least one point A from the sphere whose velocity is zero. Then its antipode point A' also has zero velocity vector. Moreover, if there is a third point with zero velocity vector, then each point of the sphere has a zero velocity vector, which is in a contradiction to our assumption. So at each moment the rotating axis is unique and well defined. Moreover, this axis changes continuously and also the vector of angular velocity \vec{w} is well defined at each moment and it is a continuous function.

Let us choose an arbitrary point of the sphere and let us denote by \vec{t} , \vec{n} and \vec{b} the orthonormal moving trihedron consisting of the tangent vector, normal vector and binormal vector. If there is a complete freedom of the space rotations on the tangent space of each point, then according to the Frenet equations, for the differential of the mapping at the considered point with respect to the moving trihedron $(\vec{t}, \vec{n}, \vec{b})$, we have the following antisymmetric matrix

$$\begin{bmatrix} 0 & k & 0 \\ -k & 0 & \tau \\ 0 & -\tau & 0 \end{bmatrix} ds$$

from the Lie algebra of $SO(3, \mathbb{R})$, where k is the curvature, τ is the torsion of the trajectory of the considered point and s is the natural parameter. On the other side this infinitesimal space rotation is not completely permitted, because the sphere is a solid body and the mutual distance between the points must be preserved. The differential of the matrix $A(t)$ with respect to the

moving trihedron $(\vec{t}, \vec{n}, \vec{b})$ is given by

$$\begin{bmatrix} 0 & k & 0 \\ -k & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} ds.$$

This means that the following orthogonal transformation

$$\begin{aligned} & \left(\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & k & 0 \\ -k & 0 & \tau \\ 0 & -\tau & 0 \end{bmatrix} ds \right) \left(\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & k & 0 \\ -k & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} ds \right)^{-1} \\ & \approx \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \tau \\ 0 & -\tau & 0 \end{bmatrix} ds \end{aligned}$$

is "not permitted". Since the matrix $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \tau ds \\ 0 & -\tau ds & 0 \end{bmatrix}$ corresponds to the vector $-\tau \vec{t} ds$, according to the basic property of the space, we come to the conclusion that at the considered moment the spin displacement is given by

$$d\vec{L} = -\tau \vec{t} ds \times \vec{r} = -\tau \vec{t} ds \times (-r\vec{n}) = r\tau \vec{b} ds, \quad (16)$$

where $r = 1/k$ is the radius of the curvature. We can write it also in the form

$$\frac{d\vec{L}}{ds} = \left(\frac{d\vec{b}}{ds} \cdot \vec{n} \right) \vec{t} \times \vec{r}. \quad (17)$$

Remark 1. Note that the formula (17) is not Lorentz covariant. Indeed, according to a moving coordinate system with a constant 3-vector of velocity \vec{u} the curve becomes $\vec{r} - \vec{u}t$ and its torsion and curvature are not the same. The choice of the coordinate system where the formula (17) holds is not precisely determined, but we a priori assume that the basic property of the space may be applied only in such an inertial system, i.e. which moves with a constant velocity with respect to the distant stars, in which the center of the osculatory sphere rests. The osculatory sphere means the sphere determined by 4 "infinitesimally close" points near the considered point. So in general case the spin velocity should be calculated for each particle of the body in different inertial system for a chosen moment. But practically as it is a case in this paper, these inertial systems coincide with the inertial system in which the barycenter rests. The calculated value at the chosen moment is considered as a global constant in the other inertial systems.

Now the spin velocity should be integrated over all points of the sphere, in order to get the averaged spin velocity of the whole sphere. We suppose that the sphere is homogeneous, i.e. with the same matter density. For the sake of simplicity at the considered moment we assume that $\vec{b} = (0, 0, 1)$, $\vec{t} =$

$(-\sin \theta, \cos \theta, 0)$, $\vec{n} = (-\cos \theta, -\sin \theta, 0)$, $\vec{r} = (\sqrt{1-z^2} \cos \theta, \sqrt{1-z^2} \sin \theta, 0)$ and $\frac{d\vec{b}}{dt} = (p, q, 0)$. Hence we obtain

$$\begin{aligned} \int \vec{V} dP &= - \int [(p, q, 0) \cdot (\cos \theta, \sin \theta, 0)] (-\sin \theta, \cos \theta, 0) \times \vec{r} dP \\ &= - \int (p \cos \theta + q \sin \theta) (-\sin \theta, \cos \theta, 0) \times \vec{r} dP \\ &= \int (p \cos \theta + q \sin \theta) (0, 0, \sqrt{1-z^2}) dP = (0, 0, 0). \end{aligned}$$

The third coordinate is zero, because for each z_0 the integration over the circle $x^2 + y^2 + z^2 = R^2$, $z = z_0$ is equal to zero.

Now let us consider finite number of points on the sphere with radius vectors \vec{R}_i , $i \in \{1, \dots, n\}$, such that their barycenter is the center of the sphere, i.e. $\sum_i \vec{R}_i = 0$. First let us prove the following statement: *If the distance between arbitrary two points among the considered n points on the sphere remains unchanged, then the spin velocity is identically equal to zero.*

According to (17) the spin velocity of the i -th body is given by

$$\begin{aligned} \vec{V}_i &= \frac{d\vec{L}_i}{dt} = \left(\frac{d\vec{b}_i}{dt} \cdot \vec{n}_i \right) \vec{t}_i \times (-r_i \vec{n}_i), \\ \vec{V}_i &= - \left(\frac{d\vec{b}_i}{dt} \cdot \vec{n}_i \right) r_i \vec{b}_i. \end{aligned} \tag{18}$$

Since the distance between any two points of the considered n points on the sphere remains unchanged, there exists a global vector \vec{b} , which is the same for each point, i.e. for each i , $\vec{b}_i = \vec{b}$. Now, using that the barycenter for all points is the coordinate origin, it is easy to deduce that $\sum_i r_i \vec{n}_i = 0$, and hence $\vec{V} = \sum_i \vec{V}_i = 0$.

In the previous statement the points may not lie on one sphere, but it is important that distances between the points remain unchanged. Consequently, *if we consider a solid body spinning around its barycenter, then the spin velocity is identically equal to zero.*

Remark 2. Note that the previous statement is true assuming that there is no gravitational field. If there is a gravitational field, the trajectories of the curves are more complicated with respect to the inertial system which should be chosen according to the remark 1. The case of a spinning body in a gravitational field will be considered in section 7.

So we shall consider a finite number n of bodies with radius vectors $\vec{R}_1, \dots, \vec{R}_n$ and masses m_1, \dots, m_n respectively, such that $\sum_i \vec{R}_i m_i = 0$, in order their barycenter to be at the coordinate origin. Suppose that these bodies with small dimensions are moving such that $\sum_i \vec{R}_i m_i \equiv 0$ and the distances between the bodies are not constants in general case. The bodies are connected by axes of constant lengths only to their barycenter which is at the coordinate

origin. The lengths $R_1 = |\vec{R}_1|, \dots, R_n = |\vec{R}_n|$ are constants, which are not necessarily equal. It is also important that the bodies are not permitted to rotate freely, but their rotations are determined by the axes which connect them with the barycenter. We may imagine that each body is attached to a spinning sphere. So the previously derived formula (16) may be applied to each body separately, because the spin velocity for a trajectory on a spinning sphere depends only on the trajectory of the body. Finally, we come to the conclusion that the acceleration of the whole system of bodies, which are connected to their barycenter, is equal to $\vec{A} = \frac{1}{M} \sum_i \frac{d\vec{V}_i}{dt} m_i$, where $M = m_1 + \dots + m_n$ and \vec{V}_i is the spin velocity of the i -th body, given by (18). In general case \vec{A} is non-zero vector, because the binormal vectors of the trajectories are not equal in general case. Hence we have a violation of the third Newton's law.

Note that if $n = 2$ or $n = 3$ in the previous consideration, the distances between the vertices are constants, and so the spin velocity is zero. Thus, if we want to get non-zero spin velocity, we should consider n bodies, where $n \geq 4$.

Note also that if R_1, \dots, R_n are not constants, then the "imagined" spheres do not exist and we have much more complicated case, where the final acceleration should be modified.

The previously described phenomena has an important applications for motion in space far from the massive bodies. In case of gravitational field the previous formula for \vec{V}_i should be corrected according to the section 7.

6 Structural invariants for the 3+3+3-model

In this section we shall verify the basic property of the space via the 6-dimensional space $S \times SR$ and the Lie group G_s , which is a subgroup of $SO(6, \mathbb{R})$. We know that the Lie group G_s has 6 coordinates and denote them by $\xi_1, \xi_2, \xi_3, \eta_1, \eta_2, \eta_3$, such that the Lie algebra is parameterized by

$$\begin{bmatrix} 0 & -d\xi_3 & d\xi_2 & 0 & d\eta_3 & -d\eta_2 \\ d\xi_3 & 0 & -d\xi_1 & -d\eta_3 & 0 & d\eta_1 \\ -d\xi_2 & d\xi_1 & 0 & d\eta_2 & -d\eta_1 & 0 \\ 0 & d\eta_3 & -d\eta_2 & 0 & -d\xi_3 & d\xi_2 \\ -d\eta_3 & 0 & d\eta_1 & d\xi_3 & 0 & -d\xi_1 \\ d\eta_2 & -d\eta_1 & 0 & -d\xi_2 & d\xi_1 & 0 \end{bmatrix}.$$

Since the mapping

$$\begin{bmatrix} C & B \\ B & C \end{bmatrix} \mapsto (C + B, C - B),$$

where B and C are antisymmetric matrices, defines an isomorphism between the Lie algebra of G_s and the Lie algebra of $SO(3, \mathbb{R}) \times SO(3, \mathbb{R})$, this implies that we have two invariants:

$$I_1 = [(d\xi_1, d\xi_2, d\xi_3) + (d\eta_1, d\eta_2, d\eta_3)]^2,$$

and

$$I_2 = [(d\xi_1, d\xi_2, d\xi_3) - (d\eta_1, d\eta_2, d\eta_3)]^2.$$

Hence

$$J_1 = \frac{I_1 + I_2}{2} = d\xi_1^2 + d\xi_2^2 + d\xi_3^2 + d\eta_1^2 + d\eta_2^2 + d\eta_3^2 \quad (19)$$

and

$$J_2 = \frac{I_1 - I_2}{4} = d\xi_1 \cdot d\eta_1 + d\xi_2 \cdot d\eta_2 + d\xi_3 \cdot d\eta_3 \quad (20)$$

are also invariants and now we will draw some conclusions.

The vector $(d\eta_1, d\eta_2, d\eta_3)$ has interpretation of space displacement, commonly denoted by (dx, dy, dz) , while the 3×3 -matrix

$$\begin{bmatrix} 0 & d\xi_3 & -d\xi_2 \\ -d\xi_3 & 0 & d\xi_1 \\ d\xi_2 & -d\xi_1 & 0 \end{bmatrix}$$

is indeed the "Frenet antisymmetric matrix" $\begin{bmatrix} 0 & k & 0 \\ -k & 0 & \tau \\ 0 & -\tau & 0 \end{bmatrix} rds$, with respect to the orthonormal frame $(\vec{t}, \vec{n}, \vec{b})$. So with respect to this orthonormal frame,

$$(d\eta_1, d\eta_2, d\eta_3) = \vec{t}ds \quad \text{and} \quad (d\xi_1, d\xi_2, d\xi_3) = (r\tau\vec{t} + kr\vec{b})ds = (r\tau\vec{t} + \vec{b})ds.$$

Hence for the invariants J_1 and J_2 we get

$$J_1 = (2 + r^2\tau^2)ds^2, \quad J_2 = r\tau ds^2.$$

Now, according to the basic property of the space, if the space rotation for angle $-\tau\vec{t}ds$ is not permitted, as in case of rotating sphere, then we have space displacement $-\tau\vec{t}ds \times (-r\vec{n}) = r\tau\vec{b}ds$. So, the new values of the vectors $(d\eta_1, d\eta_2, d\eta_3)$ and $(d\xi_1, d\xi_2, d\xi_3)$ become

$$(d\eta_1, d\eta_2, d\eta_3) = \vec{t}ds + r\tau\vec{b}ds \quad \text{and} \quad (d\xi_1, d\xi_2, d\xi_3) = \vec{b}ds.$$

Now it is easy to see that the values of J_1 and J_2 remain unchanged after this replacement, which confirm the proposed theory.

Further let us consider a circle which rotates in its plane around its center. If the particles of the circle may freely rotate around their axes according to the Thomas precession, then we have

$$(d\eta_1, d\eta_2, d\eta_3) = \vec{t}ds, \quad (d\xi_1, d\xi_2, d\xi_3) = \vec{b}ds + \frac{\vec{w}_{Th.}}{w}ds,$$

where $\vec{w}_{Th.} = \frac{-1}{2c^2}(\vec{v} \times \vec{a})$ is the Thomas precession. On the other side, if the circle is made of strong material such that the Thomas precession is not permitted to realize, then according to the basic property of the space it is

$$(d\eta_1, d\eta_2, d\eta_3) = \vec{t}ds + \vec{V}dt \quad \text{and} \quad (d\xi_1, d\xi_2, d\xi_3) = \vec{b}ds,$$

where $\vec{V} = \vec{w}_{Th.} \times \vec{r} = \frac{1}{2c^2}(\vec{v} \times \vec{a}) \times \vec{r}$. Now it is easy to verify that $J_2 = 0$ in both cases. Having in mind that $ds = vdt$, $v = rw$, $a = rw^2$ and the directions of $\vec{w}_{Th.}$ and \vec{V} , in both cases we obtain $J_1 = 1 + \left[1 - \frac{r^2 w^2}{2c^2}\right]^2$. Hence, we have again confirmation that J_1 and J_2 are really invariants. Now assume that instead of the Thomas precession we have an angular velocity which is opposite to the angular rotation of the circle. Then in the first case, when the particles of the circle may freely rotate around their axes, we have

$$(d\eta_1, d\eta_2, d\eta_3) = \vec{t}ds, \quad (d\xi_1, d\xi_2, d\xi_3) = \vec{b}ds - \vec{b}ds = 0.$$

Further if the circle is made of strong material and the rotations of the particles around their axes are not permitted, we have

$$(d\eta_1, d\eta_2, d\eta_3) = \vec{t}ds + \left(\frac{-\vec{b}ds}{r}\right) \times (-r\vec{n}) = 0 \quad \text{and} \quad (d\xi_1, d\xi_2, d\xi_3) = \vec{b}ds.$$

We notice that in both cases $J_2 = 0$ and $J_1 = 1$. This is a trivial consequence, because the circle is blocked to move, if the particles of the circle stop to rotate.

The two invariants (19) and (20) were deduced for negligible velocities. In general case the two antisymmetric matrices $C + B$ and $C - B$ should be complexified as $C + B + iT$ and $C - B + iT$, and hence we have two complex invariants

$$I_1 = [(d\xi_1, d\xi_2, d\xi_3) + (d\eta_1, d\eta_2, d\eta_3) + ic(d\theta_1, d\theta_2, d\theta_3)]^2,$$

and

$$I_2 = [(d\xi_1, d\xi_2, d\xi_3) - (d\eta_1, d\eta_2, d\eta_3) + ic(d\theta_1, d\theta_2, d\theta_3)]^2,$$

which analogously to (19) and (20), lead to the following four real invariants:

$$J_1 = d\xi_1^2 + d\xi_2^2 + d\xi_3^2 + d\eta_1^2 + d\eta_2^2 + d\eta_3^2 - c^2 d\theta_1^2 - c^2 d\theta_2^2 - c^2 d\theta_3^2, \quad (21)$$

$$J_2 = d\xi_1 \cdot d\eta_1 + d\xi_2 \cdot d\eta_2 + d\xi_3 \cdot d\eta_3, \quad (22)$$

$$J_3 = d\xi_1 \cdot d\theta_1 + d\xi_2 \cdot d\theta_2 + d\xi_3 \cdot d\theta_3, \quad (23)$$

$$J_4 = d\eta_1 \cdot d\theta_1 + d\eta_2 \cdot d\theta_2 + d\eta_3 \cdot d\theta_3. \quad (24)$$

The geometrical/physical interpretation of $(d\theta_1, d\theta_2, d\theta_3)$ means that $d\theta_1^2 + d\theta_2^2 + d\theta_3^2 = d\theta^2$, where $d\theta$ is the 1-dimensional time parameter. According to the comment at the end of section 2 (see also [13]), we have that $(d\theta_1, d\theta_2, d\theta_3)$ is orthogonal to $(d\eta_1, d\eta_2, d\eta_3)$, i.e. $J_4 = 0$. Analogously, $(d\theta_1, d\theta_2, d\theta_3)$ is orthogonal to $(d\xi_1, d\xi_2, d\xi_3)$, i.e. $J_3 = 0$. The conditions $J_3 = J_4 = 0$ mean that all points of a body which is in rest have the same temporal coordinates and also the space rotation of this body does not change the temporal coordinates of its points. Hence as a consequence we obtain that $(d\theta_1, d\theta_2, d\theta_3)$ is collinear to the unit vector \vec{n} , i.e. $(d\theta_1, d\theta_2, d\theta_3) = \vec{n}d\theta$. Analogously to $(d\xi_1, d\xi_2, d\xi_3)$ and $(d\eta_1, d\eta_2, d\eta_3)$, $d\theta$ which is related to the "speed of time"

neglecting the 6-dimensional space $S \times SR$, can be represented in the form $d\theta^2 = (\frac{ds}{c})^2 + dt^2$. Finally, we have again two invariants J_1 and J_2 , where $J_1 = d\xi_1^2 + d\xi_2^2 + d\xi_3^2 + d\eta_1^2 + d\eta_2^2 + d\eta_3^2 - c^2 dt^2 - ds^2$.

Remark 3. We considered a simple case when there is a conversion of the whole quantity $-\tau \vec{t} ds$. If only a part of this quantity is unpermitted and should be converted into spin velocity, then the accepted basic property of the space is approximately true. The exact calculation should be done using that J_1 and J_2 are invariants. Besides to the spin velocity which is parallel to the vector \vec{b} , there will appear also a slight spin velocity parallel to the unit vector of motion \vec{t} .

Note that if we have constraints for the space rotations (analogous to the section 5), and also we have constraint for the space displacement, i.e. spin velocity, such that the spin velocity is not permitted, then according to the invariant J_1 there will appear time displacement which is analogous to the space displacement. This displacement in time can be called *spin time*, which is indeed popularly called "time travel". The summand $\vec{e}\Delta t$, in (10) which represents our time is probably spin time and it is more convenient to write it in the form $c\Delta\vec{T}$, where $\Delta\vec{T}$ is collinear with the velocity and $|\Delta\vec{T}|$ is indeed the 1-parametric time. The spin time displacement changes the "speed of time" at the considered point, analogously to the Special and General Relativity. The spin time flow is almost a linear process which is a cosmological phenomena, but it is a regional effect such that the speed of time may vary in different regions in the universe.

At the end we deduce the formula for the energy obtained by a spin motion. If we neglect the friction, the energy obtained via spin motion from zero up to \vec{V} can be obtained if we integrate the spin acceleration, i.e.

$$E = \int m \frac{d\vec{V}}{dt} d\vec{r} = \int m d\vec{V} \cdot \vec{V} = \frac{1}{2} m V^2. \quad (25)$$

The spin kinetic energy is a part of the total energy, but differs from the "ordinary" kinetic energy, and the total kinetic energy is not $\frac{m}{2}(\vec{v} + \vec{V})^2$. We observe the total velocity as $\vec{v} + \vec{V}$, but the total kinetic energy is $\frac{1}{2}mv^2 + \frac{1}{2}mV^2$. This leads to some non-expecting experimental results, which can not be explained via the classical 3 + 1-approach of the space-time. For example, assume that one spinning body and one non-spinning body are thrown with the same initial velocity, i.e. $\vec{v} + \vec{V}$ is the same for both of them, then their trajectories will differ because they have different kinetic energies.

The invariant expression J_1 leads to the total kinetic energy, if we multiply this invariant with $\frac{1}{2}m\frac{1}{dt^2}$. We come also to the expression

$$(dS)^2 = ds^2 + ds^2 + r^2 \tau^2 ds^2 - c^2 dt^2 - ds^2 = ds^2 + V^2 dt^2 - c^2 dt^2, \quad (26)$$

where ds is the term for the space line element. This element dS is a generalization of the known space-time element in the Special Relativity.

7 Spinning bodies in gravitational field

The right side of (18) can easily be written using the radius vector \vec{r}_i of the i -th particle (body). It leads to the following general formula for the spin velocity

$$\vec{V} = \left[\sum_i \frac{|\vec{r}_i'|^4 (\vec{r}_i', \vec{r}_i'', \vec{r}_i''')}{|\vec{r}_i' \times \vec{r}_i''|^4} (\vec{r}_i' \times \vec{r}_i'') m_i \right] \frac{1}{\sum_j m_j}, \quad (27)$$

where we assume that $\sum_i m_i \vec{r}_i' = \vec{0}$, because the barycenter of the bodies should rest at the considered moment. If the condition $\sum_i m_i \vec{r}_i' = \vec{0}$ is not satisfied, then the following more general formula holds

$$\vec{V} = \left[\sum_i \frac{|\vec{r}_i' - \vec{u}|^4 (\vec{r}_i' - \vec{u}, \vec{r}_i'', \vec{r}_i''')}{|(\vec{r}_i' - \vec{u}) \times \vec{r}_i''|^4} ((\vec{r}_i' - \vec{u}) \times \vec{r}_i'') m_i \right] \frac{1}{\sum_j m_j}, \quad (28)$$

where $\vec{u} = \sum_i \vec{r}_i' \frac{m_i}{\sum_j m_j}$ is the velocity of the barycenter.

Now let us consider the case when the body (or system of bodies), is inside a gravitational field with acceleration \vec{g} . We shall consider two separate cases, when the body (bodies) is in free fall motion in the gravitational field, and (b) when the spinning body is put on (a horizontal) plane which is orthogonal to the vector \vec{g} .

(a) Assume that the body (bodies) is in free fall motion in the gravitational field. In this case the acceleration \vec{r}_i'' from (28) should be replaced by $\vec{r}_i'' - \vec{u}' = \vec{r}_i'' - \vec{g}$, while \vec{r}_i''' from (28) should be replaced by $\vec{r}_i''' - \vec{u}'' = \vec{r}_i''' - \vec{g}'$ and hence the spin velocity with respect to any inertial (non-rotating and not accelerated) coordinate system in the universe is given by the following formula

$$\vec{V} = \left[\sum_i \frac{|\vec{r}_i' - \vec{u}|^4 (\vec{r}_i' - \vec{u}, \vec{r}_i'' - \vec{g}, \vec{r}_i''' - \vec{g}')}{|(\vec{r}_i' - \vec{u}) \times (\vec{r}_i'' - \vec{g})|^4} ((\vec{r}_i' - \vec{u}) \times (\vec{r}_i'' - \vec{g})) m_i \right] \frac{1}{\sum_j m_j}, \quad (29)$$

where \vec{u} is the same velocity as in the formula (28). This spin velocity calculated from an inertial coordinate system is the same (neglecting terms of order c^{-2}) in any other system. In all these formulas (27), (28) and (29) it is important that each particle (body) is connected with the barycenter with an axis with a constant length. We must emphasize that in the previous formulas (27), (28) and (29) in the first and the second derivatives $\vec{r}_i' - \vec{u}$ and $\vec{r}_i'' - \vec{g}$ are not included the spin velocity and spin acceleration respectively, which are consequences indeed. For the sake of simplicity we will assume that \vec{g} is constant, i.e. $\vec{g}' = 0$.

Starting from the general formula (29), numerically can be computed the trajectory of arbitrary spinning body, if we know the law of spinning, the trajectory including spin displacement, numerically can be computed. In this section we shall consider the spacial case when

$$|(\vec{r}_i' - \vec{u}) \times \vec{g}| \ll |(\vec{r}_i' - \vec{u}) \times \vec{r}_i''|, \quad (30)$$

in order to estimate the term $|(\vec{r}_i' - \vec{u}) \times (\vec{r}_i'' - \vec{g})|^4$. This may happen if the gravitational acceleration is very small, or if the angular velocity w of the spinning body is very large. Note that if

$$|(\vec{r}_i' - \vec{u}) \times \vec{r}_i''| < |(\vec{r}_i' - \vec{u}) \times \vec{g}|,$$

then the spin velocity is much less and so it is of minor interest.

We know that the spin velocity is 0 if $\vec{g} = 0$. But if $\vec{g} \neq 0$, then according to the condition (30) the spin velocity separately for each body/particle is given by

$$\vec{V} \approx \vec{V}_I + \vec{V}_{II} + \vec{V}_{III} + \vec{V}_{IV}, \quad (31)$$

where

$$\vec{V}_I = \vec{g} \cdot (\vec{r}' \times \vec{r}''') \frac{|\vec{r}'|^4 (\vec{r}' \times \vec{r}'')}{|\vec{r}' \times \vec{r}''|^4}, \quad (32)$$

$$\vec{V}_{II} = -\vec{g} \cdot (\vec{r}' \times \vec{r}'') \frac{|\vec{r}'|^4 (\vec{r}' \times \vec{g})}{|\vec{r}' \times \vec{r}''|^4}, \quad (33)$$

$$\vec{V}_{III} = -(\vec{r}', \vec{r}'', \vec{r}''') \frac{|\vec{r}'|^4 (\vec{r}' \times \vec{g})}{|\vec{r}' \times \vec{r}''|^4}, \quad (34)$$

$$\vec{V}_{IV} = 4(\vec{r}', \vec{r}'', \vec{r}''') \frac{|\vec{r}'|^4 (\vec{r}' \times \vec{r}'')}{|\vec{r}' \times \vec{r}''|^6} [(\vec{r}' \times \vec{r}'') \cdot (\vec{r}' \times \vec{g})], \quad (35)$$

where we assumed that $\vec{u} = 0$ at the considered moment. Further these four types of spin velocities should be summed for each small body/particle. For the sake of simplicity we assume to consider a compact body which is axially symmetric and which is spinning around its axis. When we sum over all small particles of the compact body, it is convenient first to sum over the circles around the axis of the whole body and then appear some simplifications. We will omit these long calculations of integration and give the final results. The spin velocities \vec{V}_I , \vec{V}_{II} , \vec{V}_{III} and \vec{V}_{IV} separately average to the spin velocity of the whole system given by

$$\vec{V}_I = \frac{1}{w^2} \left(\frac{d\vec{b}}{dt} \cdot \vec{g} \right) \vec{b} + \frac{3}{w^3} (\vec{g} \cdot \vec{b}) \frac{dw}{dt} \vec{b} - (\vec{g} \cdot \vec{b}) \vec{b} \frac{2}{Mw^2} \int \frac{d \ln k}{dt} dm, \quad (36)$$

$$\vec{V}_{II} = 0 \quad (37)$$

$$\vec{V}_{III} = \frac{1}{2w^2} \vec{g} \times \left(\vec{b} \times \frac{d\vec{b}}{dt} \right), \quad (38)$$

$$\vec{V}_{IV} = \frac{-2}{w^2} \left(\frac{d\vec{b}}{dt} \cdot \vec{g} \right) \vec{b}, \quad (39)$$

where $\vec{b} = \frac{\vec{r}' \times \vec{r}''}{|\vec{r}' \times \vec{r}''|}$ is the same for all particles of the body with respect to the chosen coordinate system, k is the curvature of the corresponding particle of the body with respect to the coordinate system in which the barycenter rests and $M = \int dm$ is the mass of the whole body. The integral term in (36)

disappears in some special cases. For example, in case of spinning sphere this integral term is equal to 0, because $\int \ln k dm$ is "large" global constant which does not depend on time. More generally, if the considered body is axially symmetric and if its axis is at the same time spinning axis, then this integral term in (36) is also equal to 0.

Hence for the total (averaged) spin velocity we obtain

$$\begin{aligned} \vec{V} \approx & \frac{-1}{w^2} \left(\frac{d\vec{b}}{dt} \cdot \vec{g} \right) \vec{b} + \frac{3}{w^3} (\vec{g} \cdot \vec{b}) \frac{dw}{dt} \vec{b} - (\vec{g} \cdot \vec{b}) \vec{b} \frac{2}{Mw^2} \int \frac{d \ln k}{dt} dm + \\ & + \frac{1}{2w^2} \vec{g} \times \left(\vec{b} \times \frac{d\vec{b}}{dt} \right). \end{aligned} \quad (40)$$

For the sake of simplicity we assume that there is no friction, such that $w = \text{const}$. Then the last summand from the right side of (40) represents a vector of spin velocity which lies in the plane orthogonal to the gravitational vector \vec{g} . The corresponding vector of spin acceleration lies also in the same plane. We also assume that the considered spinning body is a sphere. We would like to find the projection of the spin acceleration vector to the gravitational vector \vec{g} . Hence we obtain

$$\begin{aligned} \frac{\vec{g}}{g} \cdot \frac{d\vec{V}}{dt} &= \frac{-1}{gw^2} \left(\frac{d\vec{b}}{dt} \cdot \vec{g} \right) \left(\frac{d\vec{b}}{dt} \cdot \vec{g} \right) - \frac{1}{gw^2} \left(\frac{d^2\vec{b}}{dt^2} \cdot \vec{g} \right) (\vec{b} \cdot \vec{g}) = \\ &= \frac{-1}{2gw^2} \frac{d^2}{dt^2} [(\vec{b} \cdot \vec{g})(\vec{b} \cdot \vec{g})] = \frac{-1}{2gw^2} \frac{d^2}{dt^2} (|\vec{b}|^2 |\vec{g}|^2 \cos^2 \varphi) = \\ &= \frac{-g^2}{2gw^2} \frac{d^2}{dt^2} \frac{1 + \cos 2\varphi}{2} = \frac{-g}{4w^2} \frac{d^2 \cos 2\varphi}{dt^2}, \end{aligned}$$

where $g = |\vec{g}|$ and φ is the angle between \vec{b} and \vec{g} . Hence we come to the conclusion that *the departure of the Newton acceleration \vec{g} for a spinning sphere in a gravitational field is given by*

$$\frac{\vec{g}}{g} \cdot \frac{d\vec{V}}{dt} = \frac{-g}{4w^2} \frac{d^2 \cos 2\varphi}{dt^2}, \quad (41)$$

which can be measured. If we can conduct the direction of the axis of rotation, i.e. the vector \vec{b} , then we can increase or decrease the Newton gravitational acceleration. Remember that the previous conclusion is true assuming that w is sufficiently large such that the condition (30) is satisfied. So this change of the Newton acceleration is a slight effect and the times for increasing/decreasing are short. Notice that this example implies a violation of the Principe of equivalence from the General Relativity.

(b) Now, let us consider the case when the spinning body is put on plane which is orthogonal to the vector \vec{g} . This case is not sufficiently studied, but we shall deduce some conclusions. Without loss of generality we may assume that

the spinning body is put on the xy -plane and the acceleration \vec{g} is parallel to the z -axis. For the sake of simplicity we assume that the barycenter of the spinning body is constrained to move in the xy -plane. We accept the assumption that in this case the projection of the spin velocity on the horizontal plane is just the projection of the spin velocity (40) on the horizontal plane. In other words, for determination of the spin velocity in the horizontal plane we may use the old formula (40).

Let us assume first that the unit vector \vec{b} has the form

$$\vec{b} = (\sin \varphi \cos \psi, \sin \varphi \sin \psi, \cos \varphi),$$

where φ is a constant angle. For the sake of simplicity we assume that ψ is a linear function of t , i.e. $\psi = \Omega t$, where $\Omega = \text{const.}$, such that there is no friction and assume also that the integral term in (40) is equal to 0. If we replace this value for \vec{b} into (40) we obtain

$$\vec{V} = -(-\sin \Omega t, \cos \Omega t, 0) \frac{g\Omega \sin \varphi \cos \varphi}{2w^2}, \quad (42)$$

and hence

$$\vec{A} = -(-\cos \Omega t, -\sin \Omega t, 0) \frac{g\Omega^2 \sin \varphi \cos \varphi}{2w^2}. \quad (43)$$

Now we come to the following two conclusions:

(i) *The spinning body (i.e. its barycenter) moves on a circle with the same angular velocity Ω as the angular velocity for the unit vector \vec{b} , and moreover, \vec{V} is orthogonal to the unit vector \vec{b} ;*

(ii) *The radius of the circle of motion of the spinning body is equal to*

$$R_{\text{circle}} = \frac{|\vec{V}|}{|\Omega|} = \frac{g|\sin 2\varphi|}{4w^2} \quad (44)$$

Both these conclusions experimentally can be verified.

At the end let us assume that the vector \vec{b} remains a constant and the curvature is not permitted to change, which means that $(\vec{g} \cdot \vec{b}) \vec{b} \frac{2}{Mw^2} \int \frac{d \ln k}{dt} dm = 0$. Then according to (40) the spin velocity is given by

$$\vec{V} = \frac{3}{w^3} (\vec{g} \cdot \vec{b}) \frac{dw}{dt} \vec{b}. \quad (45)$$

After simple integration we obtain that the spin displacement in a horizontal direction is equal to

$$L = \frac{3}{2} g [w_2^{-2} - w_1^{-2}] \sin \varphi \cos \varphi$$

when the angular velocity falls from w_1 to w_2 because of friction, and φ is the angle between the vector \vec{b} and the horizontal plane. So the displacement will

be largest if we assume that $\varphi = \pi/4$, and then we have displacement in the horizontal plane

$$L = \frac{3}{4}g[w_2^{-2} - w_1^{-2}], \quad (46)$$

which is easy to be measured. We emphasize that this formula is deduced for sufficiently large angular velocities w_1 and w_2 , or alternatively the dimensions of the average circular radius of the spinning body should be sufficiently large.

Obviously, a verification of the formula (46) experimentally shows that the acceleration \vec{g} is a global invariant for a considered particle, which can be measured according to formula (46). Consequently, it is not correct to say that "in a free falling shielded laboratory it is not possible to determine whether the shielded laboratory is in free falling or the shielded laboratory is far from the massive bodies", which appears in textbooks of General Relativity, when they consider the Principle of Equivalence. Indeed, the performance of the previous experiment in the shielded laboratory and after measuring the spin displacement L according to (46), we know that the shielded laboratory is far from massive bodies only if $L = 0$.

Finally, we deduce also the following important conclusion from the equations (45) and (46). Assume that a disc is placed such that $\varphi = \pi/4$, which rotates at the initial moment with angular velocity w and assume that the friction is negligible. For the sake of simplicity we assume that the biggest part of the mass of the disc is concentrated on distance r from the center of the disc. Let us accelerate the disc from 0 angular velocity up to angular velocity $w \approx 1.064\sqrt{\frac{g}{r}}$. Since the spin velocity \vec{V} is proportional to w' according to (45), we can accelerate very slowly, such that the spin velocity is almost negligible. So we input an energy E only for the kinetic energy of the rotating disc. Further let us quickly decelerate this angular velocity up to zero angular velocity ($w = 0$). In this process we can return the input kinetic energy E . Moreover, since $w' \neq 0$ means that the spin velocity \vec{V} is non-zero, the derivative of the spin velocity \vec{V} makes pressure, which is indeed *free energy*. We can repeat this process periodically many times as we wish and hence we obtain free energy. In practice the friction is always non-zero, but we should decelerate the disc very quickly such that the free energy is larger than the energy spent for the friction.

This free energy is obviously proportional to $\frac{1}{2}\vec{V}^2$. If the angular velocity changes periodically between w_1 and w_2 , where w_1 and w_2 are sufficiently large and have the same sign, then the formula (45) may be used for calculation of the obtained free energy. But in general case, when w is not so large the right side of formula (45) should be corrected by multiplication with coefficient $\lambda = \frac{|\vec{r}'_i \times \vec{r}''_i|^4}{|\vec{r}'_i \times (\vec{r}''_i - \vec{g})|^4}$. In case of $\varphi = \pi/4$, the averaged coefficient for all particles of the disc is $\lambda = \frac{1}{2\pi A^4} \int_0^{2\pi} \frac{dx}{[1 + (\cos x + \frac{1}{A})^2]^2}$, where $A = \frac{\sqrt{2}}{2} \frac{g}{rw^2}$, while the free energy is estimated to $E = 1.13mr^2((\ln w)')^2$ per one cycle, where $(\ln w)'$ is taken at the moment when $w \approx 1.064\sqrt{\frac{g}{r}}$.

This is not unique way to derive free energy using only mechanics. The free

energy may be obtained also in a gravitational field by varying of the vector \vec{b} , i.e. the axis of the spinning body, but the previous procedure is probably the simplest way to show the existence of the free energy.

8 Conclusion

Our classical view of the 3+1-dimensional space-time leads to anomaly if we consider precession of a gyroscope's axis. This anomaly disappears if we consider parallel transport of a Lorentz transformation, instead of separately transport of the velocity 4-vector and spin 4-vector. This approach naturally leads to 3+3+3-model of the universe based on the three 3-dimensional sets: velocities (V) which is homeomorphic to \mathbb{R}^3 , space (S) which is homeomorphic to $SO(3, \mathbb{R})$, and space rotations (SR) which is $SO(3, \mathbb{R})$. According to the space $S \times SR$ in this model there are no translations, but only rotations in high dimensions. As a consequence we derived new type of motion - spin velocity. It can be interpreted simply as displacement in the space. It is shown that the third Newton's law is not satisfied for this motion, and the spin velocity is not limited by c . Using the spin velocity, which is indeed non-inertial velocity, it was easy to show the violation of the third Newton's law, the law of preserving the total energy and the Principle of Equivalence from the General Relativity.

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